PGS-251021

Autonomous Examination Dec-2025

SEMESTER-I

M.Sc. (Mathematics)

Paper – I (Advanced Abstract Algebra - I)

Time : 3 Hours M.M. : 70

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

Section-A

01. Objective type questions Attempt any 10 questions out of 12 given below.

(1 x 10 = 10)

(i) Which of the following abelian group has a composition series.

(a) Group of integers (b) Group of rationals

(c) Group of integers mudulo n (d) Group of real numbers

(ii) If $|G|$ denotes order of group G. Then which of the followings implies that G is nilpotent?

(a) $|G| = 6$ (b) $|G| = 9$

(b) $|G| = 24$ (d) $|G| = 120$

(iii) If M is an R-module and A,B are R submodules of m, then

(a) $A \cap B$ is an R-submodule of M

(b) $A+B = \{a+b \mid a \in A, b \in B\}$ is an R-submodule of M

(c) $A \times B$ is an R-submodule of M

(d) All of the above

(iv) $\lambda \in F$ is called a characteristic root (or eigen value) of $T \in A$ (v) if

(a) T is singular (b) λI is singular

(c) $\lambda I - T$ is non-singular (d) $\lambda I - T$ is singular

(v) The number of non-zero diagonal entries in the smith normal form equals the

(a) Determinate (b) Rank of the matrix

(c) both (a) and (b) (d) none of the above

(vi) A sequence $(G_0, G_1, G_2, \ldots, G_n)$ of subgroups of a group G is called a subnormal series of G if …….....

(vii) If two groups H and K are solvable then $H \times K$ is also …….....

(viii) An R-module M is called free if it has a …….

(ix) A linear transformation $T \in A_F(V)$ is called nilpotent if ……...

(x) The index of nilpotency of the matrix $A = \begin{bmatrix} -2 & 1 & 1 \\ -3 & 1 & 2 \\ -2 & 1 & 1 \end{bmatrix}$ is …….....

(xi) If $T \in A(V)$ is nilpotent, then the invariants of T are Unique. (True/False)

(xii) The smith normal form can be used to classify finitely generated modules over a principal ideal domain (PID).

(True/False)

Section-B

02. Very Short answer type questions (word limit – 70-100 words) Attempt any Three questions out of Five given below.

(03 x 03= 09)

(i) Show that a group of order P^n (P-prime) is nilpotent.

(ii) Let G be a group and let G' be the commutator subgroup of G. Then show that G' is a normal subgroup of G.

(iii) Define R-module over ring R with an example.

(iv) Obtain the smith normal form and rank for the matric with integral entries $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}$

(v) Show that the abelian group generated by x_1 and x_2 subject to $2x_1=0$, $3x_2=0$ is isomorphic to Z/ (6) .

Section-C

03. Short answer type questions (word limit – 200-250 words) Attempt any Three questions out of Five given below.

(06 x 03= 18)

(i) Show that every subgroup of solvable group is solvable.

(ii) State and prove Schuler's lemma.

(iii) Prove that two nilpotent linear transformations S, $T \in A(V)$ are similar if and only if they have same invariants.

(iv) Show that if U and V be two vector spaces over a field F of dimensions m and n respectively, then Hom (U,V) is a vector space over F of dimension mn.

(v) Prove that the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent, and find its invariants and Jordan form.

Section-D

04. Long answer type questions (word limit – 500.600 words) Attempt any three questions out of five given below.

(11 x 03 = 33)

(i) State and prove Jordan-Holder theorem for finite group.

(ii) Prove that if M be a finitely generated free module over a commutative ring R, then all bases of M have the same number elements.

(iii) State and prove "Fundamental theorem of R-Homomorphism".

(iv) Let V be an n-dimensional vector space over a field F and let $T \in A(V)$ has all its characteristic roots (eigen values) in F. Then there is a basis of V in which the matrix of T is triangular.

(v) Find the invariant factors of the matrix $\begin{bmatrix} -x & 4 & -2 \\ -3 & 8-x & 3 \\ 4 & -8 & -2-x \end{bmatrix}$ over the ring Q (x). Also find the rank.

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PGS-253019
Autonomous Examination-2025
SEMESTER-III
M.Sc. (Mathematics)
Paper – III (Operation Research - I)

Time : 3 Hours M.M. : 70

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

Section-A

01. Objective type questions Attempt any 12 questions out of 10 given below. (1 x 10 = 10)

(i) The feasible region in LPP is always a …….. set.
(ii) The full form of OR is ……..
(iii) The simplex method was developed by ……...
(iv) The Hungarian method is used for ……. problem.
(v) The transportation problem aims to minimize …….
(vi) The main objective of operations Research is:
(a) Profit Reduction (b) Decision-making and optimization
(c) Employee Management (d) Data Collection
(vii) In graphical method, the number of decision variables should not exceed:
(a) 1 (b) 2
(c) 3 (d) 4
(viii) Sensitivity analysis is performed:
(a) After obtaining the optimal solution
(b) Before obtaining the optimal solution
(c) During the initial iteration
(d) Never
(ix) The main aim of parametric analysis is:
(a) To find infeasible solution
(b) To reduce number of constraints
(c) To study stability of optimal solution
(d) To simplify matrix
(x) Parametric linear programming is part of:
(a) Post – optimality analysis (b) Pre - optimization
(c) Dual Programming (d) Big – M - Method
(xi) Two – phase method avoids the use of artificial variables completly. (True/False)
(xii) The assignment problem is a special case of transportation problem. (True/False)

Section-B

02. Very Short answer type questions (word limit – 70-100 words) Attempt any Three questions out of Five given below. (03 x 03= 09)

(i) A manufacturer produces two types of products, X and Y. One unit of product requires 3 hours on machine A and 1 hour on machine B. One unit of product Y requires 2 hours on machine A and 2 hours on machine B.
The available machine hours per week are 20 hours for machine A and 30 hours for machine B. If the profit contribution per unit of X and Y are 20 rs and 30 rs respectively, Formulate this situation as a standard LPP.
(ii) Formulate the dual if the following LPP
Min $Z = x_1 + x_2 + x_3$
Sub to, $x_1 + 2x_2 + \geq 20$
$3x_1 + 2x_2 + \geq 30$
X_1, x_2 are non – negative integers
(iii) Define Operation Research and explain its scope in decision-making.
(iv) What is feasible solution in a Linear Programming Problem?
(v) Write three difference between assignment problem and transportation problem.

Section-C

03. Short answer type questions (word limit – 200-250 words) Attempt any Three questions out of Five given below. (06 x 03= 18)

(i) Solve the following L.P.P. by graphical method
Max $z = 3x_1 + 2x_2$
Sub to, $-2x_1 + x_2 + \leq 1$
$X_1 \leq 2$
$X_1 + x_2 \leq 2$
Whereas $x_1, x_2 \geq 0$
(ii) Solve by BIG –M method
Min $z = 2x_1 + 3x_2$
Sub to, $x_1 + x_2 \geq 5$

$X_1 + 2x_2 \geq 6$

$X_1, x_2 \geq 0$

(iii) Solve by simply method

Max $Z = 3x_1 + 2x_2$

Sub to, $x_1 + x_2 \leq 4$

$X_1 - x_2 \leq 2$

$X_1 \geq 0, x_2 \geq 0$

(iv) What is the difference between Big-M method and Two-phase method in LPP.

(v) Define Sensitivity Analysis and explain its importance in Operation Research.

Section-D

04. Long answer type questions (word limit – 500.600 words) Attempt any three questions out of five given below.

(11 x 03 = 33)

(i) Use two – phase method to solve the LPP

Max $Z = 5x_1 - 4x_2 - 3x_3$

Sub to, $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 + 3x_2 + 6x_2 \leq 50$

$X_1, x_2, x_3 \geq 0$

(ii) Solve the LPP by using dual simplex method

Min $Z = 3x_1 + x_2$

Sub to,

$X_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

$X_1, x_2 \geq 0$

(iii) Use the upper bound Techniques to solve the following L.P.P.

Max $Z = 3x_1 + 5x_2 + 2x_3$

Sub to, $x_1 + 2x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 23$

$0 \leq x_1 \leq 4, 2 \leq x_2 \leq 5, 0 \leq x_3 \leq 3$

(iv) A company has three factories and four warehouses. The transportation cost per unit from each factory to each warehouse is given.

Formulate the problem as a transportation model to mining the total transportation cost.

(v) What is Parametric Linear programming? Explain the method of solving it and its practical significance in decision-making.

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